Answers to Exercises for
An Introduction to Formal Logic by Richard L. Epstein

Advanced Reasoning Forum

Chapter 1

Exercises p. 5

1. See the text, p. 1.
2. a. A proposition.
   b. Not a proposition, contains an indexical.
   c. Not a proposition because not meant as true or false.
   d. Not a proposition. Commands are not true or false.
   e. Not a proposition. Quesions are not true or false.
   f. A proposition (in the context of an umpire saying it).
   g. A proposition.
   h. A proposition.
   i. A proposition.
   j. A proposition. "Anyone" and "he" are used for quantification, as we'll see later in the book.
   k. Isn't "he" an indexical? In this sentence we understand it to mean "Ralph", and the example is a proposition.
   l. A proposition.
   m. Depends on whether you think that statements that cannot be verified are OK to classify as true or false.
3. a. See the text, p. 2.
   b. See the text, p. 2.
   c. No because "Ralph" is used to mean differently in them.
   d. No, because one could be true and the other false.
4. A word or phrase whose meaning or reference depends on the circumstances of its use.
5. a. See the text, p. 3.
   b. See the text, p. 4.
   c. See the text, p. 4.
   d. The inference with that proposition as conclusion and the other(s) as premises is valid or strong.
   e. Same as (d).

Chapter 2

Exercises p. 10

1. Compound. Neither Maria nor Lee will pick up Manuel after class.
2. Compound. Both Maria and Lee have a bicycle.
3. Compound. AIDS can be contracted by touching a person infected with AIDS, and AIDS can be contracted by breathing air in the same room as a person infected with AIDS.
4. Not a proposition.
5. Compound. Antecedent: Spot barks. Consequent: Puff will run away.
   Contradictory: Spot will bark and Puff will not run away.
6. Compound. Antecedent: Dick will help Lee with his English exam.
   Consequent: Lee will take care of Spot next weekend.
   Contradictory: Dick will help Lee with his English exam but Lee will not take care of Spot next weekend.
7. An inference, not a compound proposition.
8. Compound. Antecedent: Manuel went to the basketball game.
   Consequent: Manuel either got a ride with Maria or Manuel left early in his wheelchair to get there.
   Contradictory: Manuel went to the basketball game but he didn't get a ride with Maria and he didn't leave early in his wheelchair to get there.
9. Compound (rewrite as "If you drop the gun, no one will get hurt.")
   Antecedent: You drop the gun.
   Conseqeuent: No one will get hurt.
   Contradictory: You drop the gun and someone will get hurt.
10. Compound. Conjunction made of two conditionals.
    First conditional: If Maria gets a raise, then she was on time for work for a month.
    Second conditional: If Maria is on time for work for a month, then she will get a raise.

Exercises p. 12

1. Valid. Excluding possibilities.
2. Valid. Excluding possibilities. (Implied conclusion: Spot knocked over our neighbor's trash can last night.
3. Valid. Indirect way of reasoning with conditionals.
4. Weak. Affirming the consequent.
5. Valid. Indirect way of reasoning with conditionals.
6. Weak. Denying the antecedent.

Chapter 3

Exercises p. 15

1. a. Conjunction. Conjuncts: “Ralph is a dog” “dogs bark”.
   b. Conditional. Antecedent: “Ralph is a dog”. Consequent: “dogs bark”.
   c. Negation. Negates “Cats bark”.
   d. Disjunction. Disjuncts: “Cats bark” and “dogs bark”.
   e. None of the forms.
   f. None of the forms; no formal symbol is in it.
   g. Conditional. Antecedent: “$\neg$ cats bark”. Consequent: “$\neg$ cats are dogs”.
   h. Not a compound; no formal symbol is in it.
   i. None of the forms; no formal symbol is in it.
   j. None of the forms; no formal symbol is in it.

Exercises p. 18

1. To make explicit the forms of propositions that we’ll study.
2. a. No.
   b. Yes.
   c. No.
   d. No.
   e. No.
   f. No.
   g. Yes.
3. a. $\neg p_0 \wedge p_{13} \rightarrow p_2$ length 4
   b. $(p_1 \rightarrow p_2) \wedge \neg p_2 \rightarrow \neg p_1$ length 4
   c. $(p_4 \wedge p_2) \vee \neg p_6) \rightarrow (p_7 \rightarrow p_8)$ length 4

Exercises p. 21

1. a. ((Cats are nasty $\wedge$ If Ralph is barking, then he will catch a cat) $\wedge$ Ralph is barking) $\rightarrow$ Four cats are sitting in a tree
   b. (Ralph is a dog $\wedge$ Four cats are sitting in a tree) $\rightarrow$ Four is a lucky number
   c $\neg$(Juney is barking loudly $\wedge \neg$ Juney is barking)
   d. Dogs bark $\rightarrow \neg\neg$ Dogs bark
   e. $\neg$(Bill is afraid of dogs $\wedge$ Ralph is barking) $\wedge$ $\neg$ Bill is walking quickly
   f. Bill is afraid of dogs $\wedge$ Ralph is barking $\rightarrow$ Bill is walking quickly
2. a. $((p_0) \wedge \neg((p_1)))$
   b. $((((p_{312}) \wedge (p_9)) \rightarrow (p_{317}))$
   c. $((p_4) \rightarrow (p_5))$
   d. $(((p_0) \wedge (p_3)) \rightarrow (p_9))$
   e. $(((p_1) \wedge (p_2)) \rightarrow (\neg((p_6) \rightarrow (p_{47}))))$

Chapter 4

Exercises p. 24

1 So the conditional will be true when the antecedent "doesn't apply".

2. a. Yes.
   b. Yes (think: "If you make an omelette, then you break eggs.)
   c. Yes, though it might be considered to be just an emphatic way to say "Suzy didn't pass the test."
   d. No. "because" indicates an inference.
   e. No. We'd need to take account of time as a semantic value of atomic propositions.
   f. Depends on whether we are willing to ignore time.
   g. Yes? Or do we need to take account of the subject matter of atomic propositions?
   h. Yes. Exclusive or.
   i. No. Depends on taking account of possibilities for atomic propositions.
   j. No. Depends on taking account of beliefs of atomic propositions.
   k. No. "no" is meant to apply to "one" (person), not a proposition.
   l. Yes.

3. Here are some possibilities.
   a. "Ralph is barking" T and "Cats are nasty" F,
   b. "Juney is barking" T
   c. Not possible. Must be T.
   d. Not possible. Must be T.
   e. First "dogs bark" T, second is T (that's why we don't allow the one atomic proposition to be assigned to two propositional variables).
   f. Not possible. Must be T.
   g. Not possible. Must be T.

Exercises p. 28

1. A model is the semi-formal language, a valuation, and the extension of that to all compound wffs by the truth-tables.

2. a. A formal wff is a tautology iff in every model its realization is evaluated as true.
   b. A semi-formal proposition is a tautology iff it is the realization of a wff that is a classical tautology.
   c. A proposition in ordinary English is a tautology if there is a good formalization of it that is a tautology.

4. a. Tautology.
   b. No: p T and q F.
   c. Tautology.
   d. Tautology.
   e. Tautology.
   f. No. p T.
   g. Tautology.

h. No. p F.
i. Tautology.
j. Tautology.
k. No. q F.
l. Tautology.
m. No. q T.
n. Tautology.
o. No. p F.
p. Tautology.
q. Tautology.
r. Tautology.
s. No. q T and p F.
t. No. q F and r F and pT.
u. Tautology.
v. Tautology.

<u>Exercises p. 32</u>

1. a. A collection of one or more formulas Γ and a single formula A such that given any model, if all the formulas in Γ are true in it, so too is A.
   b. Semantic consequence in classical propositional logic is how we understand "follows from" relative to our assumptions about form and meaning.
2. a. $\dfrac{A \vee B, \neg A}{B}$
   b. $\dfrac{A \rightarrow B, A}{B}$
   c. $\dfrac{A \rightarrow B, \neg B}{\neg A}$
   d. $\dfrac{A \rightarrow B, B \rightarrow C}{A \rightarrow C}$
3. a. $\dfrac{A \rightarrow B, B}{A}$
   b. $\dfrac{A \rightarrow B, \neg A}{\neg B}$
5. a. Valid.
   b. Valid
   c. Invalid
   d. Valid
   e. Valid
6. For example, (j)

   $\dfrac{A \vee (B \wedge C)}{((A \vee B) \wedge (A \vee C))}$

   $\dfrac{A \vee (B \wedge C)}{((A \wedge B) \vee (A \wedge C))}$

7. a. In every model in which A is true, A is true.
   b. If in every model A is true, then A is true in every model in which all the wffs in Γ are true.
   c. If all the wffs in Γ are true in a model, then since A is in Γ, A is true in the model.
   d. If all the wffs in Δ are true in a model, then since $\Gamma \subseteq \Delta$, all the wffs in Γ are true in the model, so since $\Gamma \vDash A$, A is true in the model.
   e. Given a model in which all the wffs in Γ are true, it follows that A is true since $\Gamma \vDash A$. Since $A \vDash B$, B is true in the model, too.
   f. Suppose we have a model in which all the wffs in $\Gamma \cup \Delta$ are true. Then since $\Gamma \vDash A$, A is true in the mode. Since A is true in the model and $\Delta \cup \{A\} \vDash B$, it follows that B is true in the model.
   g. Given a model in which all the wffs in Γ are true, it follows since $\Gamma \vDash A_1, \ldots, \Gamma \vDash A_n$ that B is true in the model since $\Gamma \cup \{A_1, \ldots, A_n\} \vDash B$.

<u>Exercises p. 34</u>

2. Use the equivalences from Theorem 2.
3. Given any wff that does not use $\neg$, if all the propositional variables in it are T, then the entire wff will be T. So you can't get F.

Chapter 5

Exercises p. 41

1. a. ¬ (A∧ B)
   b. A→ ¬ B
   c. A→ B
2. a. ¬ (7 is even)
   b. ¬ (7 is even) ∧ ¬ (7 is odd)
   c. 7 is even → ¬ (7 is odd)
   d. Not formalizable.
   e. logic is hard → ¬ (art history is hard)
   f. (Tom and Dick will go to the party on Saturday) ∨ Suzy will drive
   g. the play is sold out → (Zoe will stay home ∧ Dick will meet us tonight) (could move the parentheses if context demanded)
   h. ¬ (Manuel will go to the dance) → Dick will drive
   i. Not formalizable.
   j. Not formalizable.
   k. Anubis eats ∧ Anubis sleeps ∧ Anubis barks at night ("you know" is just a rhetorical flourish)
   l. you make an omelette → you break eggs
   m. Not formalizable. (This talks about occasions on which acid and water are mixed, and we can't take account of time.)
   n. (Ralph is a puppet ∧ Ralph barks) → ¬ Ralph is a puppet
3. a. cat owners homes have fleas → cats are nasty.
   cat owners homes smell bad → cats are nasty
   cat owners homes have fleas ∨ cat owners homes smell bad
   therefore, cats are nasty
   Valid.
   b. strawberries are red → some color-blind people cannot see strawberries among their leaves
   Strawberries are red.
   So: some color-blind people cannot see strawberries among their leaves.
   Valid.
   c. the students are happy ↔ ¬ (a test is given)
   the students are happy → professor feels good
   the professor feels good → ¬ (the professor feel like lecturing)
   ¬ (the professor feels like lecturing) → a test is given.
   Therefore: ¬ (the students are happy)
   Valid.
   d. Not formalizable. Valid but not any forms we recognize.
   e. Tom is Polish ∧ ¬ (Tom is from New York ∨ Tom is from Virginia)
   Tom is from Syracuse → (Tom is from New York ∨ Tom is from Virginia)
   Therefore: ¬ (Tom is from Syracuse)
   Valid. "Tom is Polish" doesn't matter.

f. Ralph is a cat → Ralph meows
¬ (Ralph is a cat)
Therefore: ¬ (Ralph meows)
Invalid. Denying the Antecedent.

g. you know some logic → (you are very bright ∨ you study very hard)
you study very hard
you are very bright
Therefore: you know some logic
Invalid. Affirming the consequent.

h. the government is going to spend less on health and welfare.
the government is going to spend less on health and welfare→
(the government is going to cut the Medicare budget ∨
the government is going to slash spending on housing.
the government is going to cut the medicare budget →
the elderly will protest
the government is going to slash spending on housing →
advocates for the poor will protest
Therefore, the elderly will protest or advocates for the poor will protest
Valid.

i. Valid.

j. Valid.

k. the moon is made of green cheese ∨ 2 + 2 = 5
¬ (the moon is made of green cheese)
Therefore, 2 + 2 = 5.
Valid.

l. Not formalizable.

Chapter 6

Exercises p. 46

1. So we can isolate our assumptions about the meanings of the connectives not by appeal to meanings but to form only.
   So we can have another way to derive tautologies.
   To understand the process of proving.
2. There is a sequence $A_1, \ldots, A_n$, where A_n is A and each A_i is either an axiom or is a derived from one or more of the preceding A_j's by one of the rules.
3. a. Use the truth-tables.
   b. Proof by induction on the length of a proof of A. If the length of the proof is 1, then A is an axiom, and so true in every model. Suppose that we have the proposition for every wff of length n, and $A_1, \ldots, A_{n+1}$, is a proof of A; that is, A is A_{n+1}. If A an axiom, then A is true in every model. Otherwise, for some $j, k \leq n$, A_j is $A_k \to A$. Since $A_k \to A$ and A_k are true in every model (by induction) it follows that A is true in every model.
4. a. (i) $D \to D$ (insert the proof from p. 45)
   (ii) $(D \to D) \to ((C \to (D \to D))$ Axiom 2
   (iii) $C \to (D \to D)$ *modus ponens* on (i) and (ii)
   b. (i) $\neg C \to (C \to \neg\neg C)$ Axiom 1
   (ii) $\neg\neg C \to (C \to \neg\neg C)$ Axiom 2
   (iii) $[\neg C \to (C \to \neg\neg C)] \to [(\neg\neg C \to (C \to \neg\neg C)) \to (C \to \neg\neg C)]$
   (iv) $(\neg\neg C \to (C \to \neg\neg C)) \to (C \to \neg\neg C)$ *modus ponens* on (i) and (iii)
   (v) $C \to \neg\neg C$ *modus ponens* on (ii) and (iv)

Exercises p. 48

1. English supplemented with technical notions.
   The metalanguage.
2. a. There is a sequence $A_1, \ldots, A_n$, where A_n is A and each A_i is an axiom, or is in Σ, or is a derived from one or more of the preceding A_j's by one of the rules.
   b. In the proof of A from Σ, if A_j is in Σ, then insert at that point the proof of A_j.
3. a. A collection of wffs Σ such that if $\Sigma \vdash A$, then A is in Σ.
   b. If Γ is a theory and A is an axiom, then $\Gamma \vdash A$, since $\vdash A$, hence A is in Γ.
   c. Σ is consistent iff there is no A such that both A and $\neg A$ are in Σ.
   d. Σ is complete iff for every A, one of A or $\neg A$ is in Σ.
   e. Σ is complete and consistent iff there is a model such that Σ is exactly the wffs true in that model.
4. Let Σ be the wffs true in M. Since either A is true in M or $\neg A$ is true in M, one of A and $\neg A$ is in M. For no A are both A and $\neg A$ true in M, so not both A and $\neg A$ can be in M. To show that if $\Sigma \vdash A$ then A is in Σ, we proceed by

induction on the length of a proof of A. It's true if the length is 1. Suppose that $A_1, \ldots, A_{n+1}$, is a proof of A; that is, A is A_{n+1}. If A is an axiom or is in Σ, then we are done. Otherwise, for some $j, k \leq n$, A_j is $A_k \rightarrow A$. Since $A_k \rightarrow A$ and A_k are in Σ, that is they are true in M, A is true in M, that is, A is in Σ.

5. 1. By definition of proof.
   2. By definition of proof.
   3. By definition of proof
   4. Whatever proof there is from Γ is also a proof from Δ, since every wff in Γ is in Δ.
   5. Let $A_1, \ldots, A_n = B$ be a proof of B from A. Let $C_1, \ldots, C_m = A$ be a proof of A from Γ. Then $C_1, \ldots, C_m, A_1, \ldots, A_n$ is a proof of B from Γ.
   6. Proceed as in (5).
   7. Left to you.

Chapter 8

Exercises p. 59

1. Things, the World, and Propositions
   The Distinguishability of Things
2. A word that is meant to pick out a specific thing.
3. I've left off quotation marks because I think it's clear what's meant as a part of speech.
   a. predicate: (—went to see —)
      names: Dick, Suzy
   b. predicates: (— is a dog) (— is a puppet)
      names: Ralph
   c. predicate: (— hit —)
      names: Ralph, Juney
   d. predicate: (— read *An Introduction to Formal Logic*)
      names: Ralph
      If you think that books (not copies of books) are individual things, you could parse the example as:
      predicate: (— read —)
      names: Ralph, *An Introduction to Formal Logic*
   e. predicate: (— is less than —)
      names: 8, 9
      This is to assume that numerals are names of things: numbers.
   f. predicate: (— is in —)
      names: Paris, France
      That's to take "Paris" and "France" as names of individual things. ??
   g. predicates: (— was sick) (— was tired)
      names: Horatio
   h. predicate: (— is a puppet)
      name: Juney
   i. ???? Is the set of natural numbers a thing?
   j. Can't parse it becaues "green" and "gray" aren't names and aren't being used as predicates here.
4. See the discussion on p. 56.
5. a. $\forall$ meant to formalize "for all", "each and every"
   b. $\exists$ meant to formalize "there exists", "there is at least one"
6. a. $\exists x \, \exists y$ ((— is a dog) $(x) \wedge$ (— is a woman) $(y) \wedge$ (— loves —) (x, y))
   b. $\forall x \, \exists y$ ((— is the biological mother of —) (x, y))
      $\exists x \, \exists y$ ((— is the biological mother of —) (x, y))

Chapter 9

<u>Exercises p. 66</u>

1. In an open proposition there is at least one variable that is used as a temporary name but we are not told what it is a temporary name of.

2. a. categorematic: — is a dog Ralph — is a cat — likes —
   syncategorematic: $\forall$ x_1 $\wedge$ $\neg$ $\rightarrow$
   b. categorematic: — knew — Socrates Plato — is dead
   syncategorematic: $\rightarrow$
   c. categorematic: — belongs to — Arf — was made in America
   syncategorematic: $\forall$ x_2 $\rightarrow$
   d. catgorematic: — belongs to the United Nations — has a president
   syncategorematic: $\forall$ x_6 $\rightarrow$
   Note: "United Nations" is not a categorematic part of this because it is not separate from the predicate; it is not being treated as a name.

4. a. Not an unabbreviated wff.
   b. Not an unabbreviated wff. The predicate symbol is for binary, but only one term is used.
   c. i. Atomic.
   ii. Neither.
   d. Not an unabbreviated wff: $\forall x_1$ is used but x_1 does not appear free in what follows.
   e. i. Not atomic.
   ii. Existential.
   f. i. Not atomic.
   ii. Neither.
   g. Not unabbreviated (missing outermost parentheses).

5. a. $(\forall x_1$ ((— is a dog) (x_1)))
   length 2
   b. $(\forall x_1$ $(\exists x_2$ (((— is a dog) (x_1)) $\rightarrow$
   (((— is a cat) (x_2)) $\wedge$ ((— hates —) (x_1, x_2)))))))
   length 4

6. a. i. No quantifier.
   ii. No variables.
   iii. Closed, atomic.
   iv. $P_1(c_8)$
   b. i. No quantifier.
   ii. The single occurrence of x_1 is free.
   iii. Open, atomic.
   iv. $\forall x_1\, P_8(x_1)$
   c. i. The scope of $\exists x_1$ is $P_1(x_1)$.
   ii. The single occurrence of x_1 after the quantifier is bound.
   iii. Closed, not atomic.
   iv. Same as original.

d. i. Scope of $\forall x_1$ is $(\neg P_2(x_1) \vee P_1(x_3))$.
Scope of $\exists x_3$ is $P_4(x_1, x_3)$
ii. The first occurence of x_1 after the quatifier is bound, second is free.
The first occurrence of x_3 is free.
iii. Open, not atomic.
iv. $\forall x_1 \forall x_3 [\forall x_1 (\neg P_2(x_1) \vee P_1(x_3)) \rightarrow \exists x_3 P_4(x_1, x_3)]$

e. i. Scope of $\forall x_2$ is $P_1(x_1, x_2, x_3)$.
Scope of $\exists x_2$ is $\exists x_3 P_1(x_1, x_2, x_3)$.
Scope of $\exists x_3$ is $P_1(x_1, x_2, x_3)$.
ii. All occurences of x_1 are free.
First occurrence of x_3 is free.
iii. Open, not atomic.
iv. $\forall x_1 \forall x_3 (\forall x_2 P_1(x_1, x_2, x_3) \rightarrow \exists x_2 \exists x_3 P_1(x_1, x_2, x_3))$

f. i. No quantifier.
ii. Both occurrences of x_1 are free.
iii. Open, not atomic.
iv. $\forall x_1 (P_1(x_1) \vee \neg P_1(x_1))$

g. i. Scope of $\forall x_1$ is $P_1(x_1)$.
Scope of $\exists x_2$ is $P_2(x_2)$.
ii. No occurrence of a variable is free.
iii. Closed, not atomic.
iv. Same as original.

7. a. i. The scope of $\exists x_1$ is (— is a woman) (x_1).
ii. The first occurrence of x_1 is free. The occurrence of x_2 is free.
iii. Open, not atomic.
iv. $\forall x_1 \forall x_2$ [(— is an uncle of —) $(x_1, x_2) \vee \exists x_1$ (— is a woman) (x_1)]

b. i. The scope of $\exists x_1$ is (— is a dog) (x_1).
ii. No variable is free.
iii. Closed, not atomic.
iv. Same as original.

c. i. The scope of the first $\forall x_2$ is $\neg$ (— is a father) (x_2).
The scope of $\exists x_2$ is ((is related to —) $(x_2$, Ralph)).
ii. No free variable.
iii. Closed, not atomic.
iv. i. Same as original.

d. i. The scope of $\forall x_1$ is
$\forall x_2$ ((— is a dog —) $(x_1) \vee$ (— is a cat) $(x_2) \rightarrow$ (— barks) (Juney)).
The scope of $\forall x_2$ is
((— is a dog —) $(x_1) \vee$ (— is a cat) $(x_2) \rightarrow$ (— barks) (Juney)).
ii. No variable is free.
iii. Closed, not atomic.
iv. Same as original.

Chapter 10

<u>Exercises p. 74</u>

1. A name picks out one and only one thing.
2. a. We take a variable, say *x*, and say that it refers to the figure, and "(— is round) (*x*)" is true.
   b. We take two variables, say *x* and *y*, and say that *x* stands for the triangle and *y* stands for the circle, and "(— stands to the left of) (*x y*)" is true.
   c. We take three variables, say *z*, *y*, *z*, and say that *x* stands for the triangle and *y* stands for the circle, and *z* stands for the square, and "(— and — are larger than —) (*x*, *y*, *z*)" is true.
   d. We take two variables, say *x* and *y*, and say that *x* stands for Marilyn Monroe and *y* stands for Donald Trump, and then "(— is more honest than —) (*x*, *y*)" is true.
3. a. The truth-value of predications using it does not depend on which name(s) you use for the objects you mean to apply it to.
   c. If we didn't, we'd have to take account of some further semantic value of names.
4. a. Yes.
   b. Any one of them serves to explain what we mean that "— is a dog" is true of Ralph.
   c. Because *x* is not meant to stand for the name "Ralph" but for Ralph himself.

<u>Exercises p. 77</u>

1. To make it paradoxical you need to assume that up to the moment it was uttered all Cretans had lied all the time; if not, it is false. But that's equally perplexing, for simply by saying that one sentence Epimenides apparently ensured the existence of another Cretan.
   A better paradox is: "What I'm now saying is false."
2. a. To avoid dealing with self-referential paradoxes.
3. We have no way to describe what we mean by picking out any thing to be the reference for a variable, not even any way to conceive of that (see Appendix 3).
4. Without at least one thing, the entire notion of predication makes no sense. We reason about whether something exists only with respect to what does exist.

Chapter 11

Exercises p. 89

1. $\nu(\exists x\, ((\text{— is a dog})(x) \vee \neg((\text{— is a dog})(x))\,)) = \mathsf{T}$
   iff there is some σ such that $\nu_\sigma(\,(\text{— is a dog})(x) \vee \neg((\text{— is a dog})(x))\,)) = \mathsf{T}$
   iff there is some σ such that $\nu_\sigma(\,(\text{— is a dog})(x)\,) = \mathsf{T}$
   or $\nu_\sigma(\neg((\text{— is a dog})(x))\,)) = \mathsf{T}$
   iff there is some σ such that $\nu_\sigma(\,(\text{— is a dog})(x)\,) = \mathsf{T}$
   or $\nu_\sigma(\,(\text{— is a dog})(x)) = \mathsf{F}$
   Any σ will do since either $\nu_\sigma(\,(\text{— is a dog})(x)) = \mathsf{T}$ or $\nu_\sigma(\,(\text{— is a dog})(x)) = \mathsf{F}$.

2. a. $\nu(\forall x\, ((\text{— loves Juney})(x) \vee \neg(\,(\text{—has a heart})(x))\,) = \mathsf{T}$
   iff for all σ, $\nu_\sigma(\,(\text{— loves Juney})(x) \vee \neg(\,(\text{—has a heart})(x)\,) = \mathsf{T}$
   iff for all σ, $\nu_\sigma(\,(\text{— loves Juney})(x)\,) = \mathsf{T}$ or $\nu_\sigma(\neg(\,(\text{—has a heart})(x)\,) = \mathsf{T}$
   iff for all σ, $\nu_\sigma(\,(\text{— loves Juney})(x)\,) = \mathsf{T}$ or $\nu_\sigma((\text{—has a heart})(x)\,) = \mathsf{F}$
   Note that this is a tautology.

   b. $\nu(\exists x(\,(\text{— is a cat})(x) \vee (\text{— is a dog})(\text{Ralph})\,)\,) = \mathsf{T}$
   iff for some σ, $\nu_\sigma(\,(\text{— is a cat})(x) \vee (\text{— is a dog})(\text{Ralph})\,)\,) = \mathsf{T}$
   iff for some σ, $\nu_\sigma(\,(\text{— is a cat})(x)\,) = \mathsf{T}$ or $\nu_\sigma(\,(\text{— is a dog})(\text{Ralph})\,)\,) = \mathsf{T}$

   c. $\nu(\forall x\, ((\text{— is a dog})(x) \rightarrow \neg(\,(\text{— eats grass})(x))\,) = \mathsf{T}$
   iff for all σ, $\nu_\sigma(\,(\text{— is a dog})(x) \rightarrow \neg(\,(\text{— eats grass})(x))\,) = \mathsf{T}$
   iff for all σ, if $\nu_\sigma(\,(\text{— is a dog})(x) = \mathsf{T}$, then $\nu_\sigma(\neg(\,(\text{— eats grass})(x))\,) = \mathsf{T}$
   iff for all σ, if $\nu_\sigma(\,(\text{— is a dog})(x)\,) = \mathsf{T}$, then $\nu_\sigma(\,(\text{— eats grass})(x)\,) = \mathsf{F}$

   d. $\nu(\forall x\, \exists y\, (\,(\text{— is the father of —})(x, y))\,) = \mathsf{T}$
   iff for all σ, $\nu_\sigma(\exists y\, (\,(\text{— is the father of —})(x, y))\,) = \mathsf{T}$
   iff for all σ, there is some $\tau \sim_x \sigma$, $\nu_\tau(\,(\text{— is the father of —})(x, y))\,) = \mathsf{T}$

   e. $\nu(\exists y\, \forall x\, (\,(\text{— is the father of —})(x, y))\,) = \mathsf{T}$
   iff for some σ, $\nu_\sigma(\forall x\, (\,(\text{— is the father of —})(x, y))\,) = \mathsf{T}$
   iff for some σ, for all $\tau \sim_y \sigma$, $\nu_\tau(\,(\text{— is the father of —})(x, y))\,) = \mathsf{T}$

   f. $\nu(\exists x\, \forall y\, (\,(\text{— is the father of —})(x, y)\,)\,) = \mathsf{T}$
   iff for some σ, $\nu_\sigma(\forall y\, (\,(\text{— is the father of —})(x, y)\,)\,) = \mathsf{T}$
   iff for some σ, for all $\tau \sim_x \sigma$, $\nu_\tau(\,(\text{— is the father of —})(x, y)\,)\,) = \mathsf{T}$

   g. $\nu(\forall x\, \forall y\, (\,(\text{— is the father of —})(x, y) \vee \exists y\, (\text{—is a clone})(y))\,) = \mathsf{T}$
   iff for all σ, $\nu_\sigma(\forall y\, (\,(\text{— is the father of —})(x, y) \vee$
   $\exists y\, (\text{—is a clone})(y)\,) = \mathsf{T}$
   iff for all σ, for all $\tau \sim_x \sigma$, $\nu_\tau(\,(\text{— is the father of —})(x, y) \vee$
   $\exists y\, (\text{—is a clone})(y)\,) = \mathsf{T}$
   iff for all σ, for all $\tau \sim_x \sigma$, $\nu_\tau(\,(\text{— is the father of —})(x, y)\,) = \mathsf{T}$ or
   $\nu_\tau(\exists y\, (\text{—is a clone})(y)\,) = \mathsf{T}$
   iff for all σ, for all $\tau \sim_x \sigma$, $\nu_\tau(\,(\text{— is the father of —})(x, y)\,) = \mathsf{T}$ or
   for some γ, $\nu_\gamma(\,(\text{—is a clone})(y)\,) = \mathsf{T}$

   h. $\nu(\forall x \neg(\exists y\, (\text{— loves —})(x, y) \wedge \neg\exists y\, (\text{— loves —})(x, y))\,) = \mathsf{T}$
   iff for all σ,
   $\nu_\sigma(\neg(\exists y\, (\text{— loves —})(x, y) \wedge \neg\exists y\, (\text{— loves —})(x, y))\,) = \mathsf{T}$

iff for all σ, $\mathsf{v}_\sigma(\neg\ (\exists y\,(\text{— loves —})\,(x, y) = \mathsf{T}$
and $\mathsf{v}_\sigma(\neg\ \exists y\,(\text{— loves —})\,(x, y))\,) = \mathsf{T}$

iff for all σ, $\mathsf{v}_\sigma(\ \exists y\,(\text{— loves —})\,(x, y) = \mathsf{F}$
and $\mathsf{v}_\sigma(\exists y\,(\text{— loves —})\,(x, y))\,) = \mathsf{F}$

iff for all σ, there is some $\tau \sim_x \sigma$, $\mathsf{v}_\tau\,(\ \exists y\,(\text{— loves —})\,(x, y)\,) = \mathsf{F}$
and there is some $\gamma \sim_x \sigma$, $\mathsf{v}_\gamma\,(\,(\text{— loves —})\,(x, y)\,) = \mathsf{F}$

3 a. $\mathsf{v}(\ \exists x\,\exists y\,((\text{— is a dog})\,(x) \wedge (\text{— is a cat})\,(y) \rightarrow (\text{— hates —})\,(x, y)\,) = \mathsf{T}$

iff for some assignment of references σ,
$\mathsf{v}_\sigma(\ \forall y\,((\text{— is a dog})\,(x) \wedge (\text{— is a cat})\,(y) \wedge (\text{— hates —})\,(x, y)\,) = \mathsf{T}$

iff for some assignment of references σ, and any assignment of references τ such that $\tau \sim_x \sigma$,
$\mathsf{v}_\tau(\,(\text{— is a dog})\,(x) \wedge (\text{— is a cat})\,(y) \rightarrow (\text{— hates —})\,(x, y)\,) = \mathsf{T}$

iff for some assignment of references γ,
$\mathsf{v}_\gamma(\,(\text{— is a dog})\,(x) \wedge (\text{— is a cat})\,(y) \rightarrow (\text{— hates —})\,(x, y)\,) = \mathsf{T}$
because if it is for all σ andτ such that $\tau \sim_x \sigma$, that can only be
if it is so for all γ, since the collection of assignments is complete

b. $\mathsf{v}(\ \forall x\,\forall y\,((\text{— is a dog})\,(x) \wedge (\text{— is a cat})\,(y) \rightarrow (\text{— hates —})\,(x, y)\,) = \mathsf{T}$

iff for any assignment of references σ,
$\mathsf{v}_\sigma(\ \forall y\,((\text{— is a dog})\,(x) \wedge (\text{— is a cat})\,(y) \wedge (\text{— hates —})\,(x, y)\,) = \mathsf{T}$

iff for any assignment of references σ, and any assignment of references τ such that $\tau \sim_x \sigma$,
$\mathsf{v}_\tau(\,(\text{— is a dog})\,(x) \wedge (\text{— is a cat})\,(y) \rightarrow (\text{— hates —})\,(x, y)\,) = \mathsf{T}$

iff for any assignment of references γ,
$\mathsf{v}_\gamma(\,(\text{— is a dog})\,(x) \wedge (\text{— is a cat})\,(y) \rightarrow (\text{— hates —})\,(x, y)\,) = \mathsf{T}$
because if it is for all σ andτ such that $\tau \sim_x \sigma$, that can only be
if it is so for all γ, since the collection of assignments is complete

4. When you write out the evaluation, you'll see that this just follows from how we interpret ∀ and ∃.

5. a. Not valid.
b. Valid.
c. Valid.

Chapter 12

Exercises p. 96

1. a. Invalid. False in any model in which there is a dog.
   b. Invalid. False in any model in which there is no dog.
   c. Valid. Theorem 4.
   d. Invalid. False in any model in which the universe is all cats. (But surely that universe should have litter boxes, too.)
   e. Valid. By PC.
   f. Valid. By PC.
   g. Invalid. Take a model with universe all physical objects including planets, stars, galaxies. The antecedent is (likely) true, but the consequent can't be true because nothing is heavier than itself.
   h. Valid. Theorem 3.
   i. Valid. Theorem 4.

2. a. $\forall x$ [(— is a cat) $(x) \rightarrow$ (— stinks) (x)]
   $\rightarrow$ [$\forall x$ (— is a cat) $(x) \rightarrow \forall x$ (— stinks) (x)]
   Valid. Theorem 5. (Hey, if you want an exercise that's favorable to cats, make up your own.)
   b. Invalid. False in a model in which there are cats as well as dogs in the universe, and some cat does not stink.
   c. Valid. Theorem 5.
   d. Invalid. False in a model in which there is a dog and which there is a thing that stinks (e.g., a cat), but in which (of course) no dog stinks.
   e. Valid. Theorem 6.
   f. Valid. Theorem 6.
   g. Invalid. Take a model with universe all cats and cars in 2019 with the "obvious" truth-values.
   h. Invalid. Could have a model in which there is a fox that barks and no dog barks.

3. By Theorem 3 of Chapter 11, $\upsilon_\sigma(\forall \ldots (A \leftrightarrow B) \rightarrow (C(A) \leftrightarrow C(B))) = \mathsf{T}$.
   So by Theorem 5 (repeated, using induction on the number of variables in the universal closure prefix), $\upsilon_\sigma(\forall \ldots (A \leftrightarrow B) \rightarrow \forall \ldots (C(A) \leftrightarrow C(B))) = \mathsf{T}$.
   so by Theorem 5 $\upsilon_\sigma(\forall \ldots (A \leftrightarrow B) \rightarrow (\forall \ldots C(A) \leftrightarrow \forall \ldots C(B))) = \mathsf{T}$.

Exercises p. 100

1. a. (— is an uncle of —) (y, Ralph)
   b. (— is an uncle of —) (y, y)
   c. (— is an uncle of —) (y, y)
   d. (— is an uncle of —) (y, Ralph) (same as original)
   e. $\exists y$ ((— is an uncle of —) (y, y))
   f. $\forall z$ ((— is an uncle of —) (z, y) $\rightarrow$ (— is an uncle of —) (y, y))
   g. $\forall x$ (— is an uncle of —) (x, y) (same as original)
2. a. (— is an uncle of —) (y, y) $\vee$ $\exists x$ (— is a woman) (x)
   b. y is not free for x
   c. y is not free for x
   d. y is not free for x
   e. (— loves —) (y, y) $\vee$ $\exists z\, \exists y$ ($\neg$ (— loves —) (y, z))
   f. (— loves —) (y, y) $\vee$ $\exists z\, \exists x$ ($\neg$ (— loves —) (z, x)))
3. a. Valid. Theorem 7.b
   b. Invalid.
   c. Valid. Theorem 8
   d. Valid. Theorem 8
   e. Valid. Theorem 8.b
   f. Valid. Existential Generalization
   g. Invalid, could have a model in which everything is loved by someone else but no one loves himself or herself.
   h. Invalid. Same model as for (g).
   i. Invalid. Same model as for (g).
   j. Valid, but not from our theorems.
   k. Valid, but not from our theorems.

Chapter 14

Exercises p. 115

1. If we don't, then we're committed to reasoning only under the assumptions of classical predicate logic and we will not be able to see the limitations of those assumptions or compare them to other ways of encountering the world.
2. a. $\forall x$ ((— is a wombat) (x) → (— squeaks) (x))

   b. $\exists x$ ((— is a wombat) (x) ∧ (— squeaks) (x))

   c. Not formalizable until we have more tools in Chapter 16.

   d. (— is a dog) (Ralph) → $\exists x$ ((— is a puppet) (x) (— is a dog) (x))

   e. (— is taller than —) (Dick, Suzy)

   f. $\forall x$ [(— is a person) (x) → (— taller than —) (x, Spot)]

   g. $\forall x$ [(— is a man) (x) →
   $\exists y$ ((— taller than —) (x, y) ∧ (— is a woman) (y)]

   h. $\forall x$ [(— is a person) (x) →
   $\exists y$ ((— knows —) (x, y) ∧ (— is a person) (y) ∧ (— is rich) (y))]

   i. $\exists x$ [(— is a clown) (x) ∧ ¬ (— is happy) (y)]

   j. $\forall x$ [(— is a dog) (x) → (— is a canine) (x)]

   k. $\exists x$ [(— is a dog) (x) ∧ ¬ (— is happy) (y)]

   l. ¬ (— respected —) (Zeke, Zoe)
   If you think that "disrespected" means more than just "not respected" then see Example 17.

Exercises p. 119

1. If we don't, then we're doing analysis before formalization. Logic is concerned with form, not the meaning of words.
2. I'd like to see yours.
3. When we cannot formalize a proposition or inference because it depends on the relation of meanings of the words, we can take a closed wff(s) that relates the words and then formalize relative to that (those) being true in a model.
4. Because we cannot separate it from the common noun (predicate) that it is modifying.
5. Because we cannot separate if from the verb (predicate) that it is modifying.
6. a. Not formalizable. "smart" is a relative adjective. The proposition is true, but Harold was really stupid compared to my dog Anubis.

   b. $\forall x$ [((— is a horse) (x) ∧ (— neighs) (x)) → (— is healthy) (x)]

   c. ¬ $\exists x$ [(— is a person) (x) ∧ $\exists y$ ((— knows —) (x, y) ∧
   (— is a sheep) (y) ∧ (— is green) (y))]

   d. $\exists x$ [((— is a dog) (x) ∧ $\exists y$ ((— is owned by —) (y, x) ∧
   (— is a woman) (y) ∧ $\forall z$ ((— is a mailman) (z) → (— barks at —) (x, z)]

Exercises p. 135

1. Use Theorem 3 of Chapter 12 and PC.
2. a. $\forall x$ [(— is a bear) $(x) \to$ (— growls) (x)]
   This is to understand the sentence as about an essential quality of bears.

   b. $\forall x$ [(— is a bear) $(x) \to$ ((— growls) $(x) \vee$ (— roars) (x))]

   c. $\forall x$ [((— is a dog) $(x) \wedge \neg$ (— is friendly) (x)) $\to$ (— is wild) (x)]

   d. $\forall x$ [((— is a scout) $(x) \to$ (— is a reverent) (x)]

   e. Not formalizable: "present" means here and now, and we can't formalize times or locations. Parity of form with the last example is violated.

   f. $\exists x$ [(— is a bone) $(x) \wedge$ (— gave —, —) (Dick, Spot, x)]

   g. Not formalizable: "food" is a mass term.

   h. Not formalizable: depends on taking "heavily" as an adverb.
   Cannot take the predicate to be "— is snoring heavily" because then can't respect: "Ralph was snoring heavily, therefore Ralph was snoring".

   i. Not formalizable: "snoring" is used as a mass or process term.

   j. $\forall x$ [(— is an alcoholic) $(x) \to$
   $\forall y$ ((— is a car) $(y) \to \neg$ (— can drive —) (x, y)]
   But it's not clear if this is meant as a rule ("can't" = "not allowed") or about the physical capacity of alcoholics. Expanding the predicate to make that clear would be acceptable.

   k. $\forall x$ [(— is a scholar) $(x) \wedge \exists y$ ((— a dog) $(y) \wedge$ (— has_p —) $(x, y) \to$
   $\forall z$ ((— is a cat) $(z) \to \neg$ (— owns —) (x, z)]
   Relative to $\forall x \forall y$ [(— owns —) $(x, y) \to$ (— has_p —) (x, y)]
   Not $\leftrightarrow$ because no one owns a cat.
   The plural doesn't mean that a scholar doesn't own more than one cat.

   l. This has to be taken as a general rule, but one that depends on time, for it means that a dog doesn't catch the cat at any time when the dog is hungry. So we can't formalize it.

   m. The problem here is that "eats mice" means habitually, not that there is at least one mouse that the cat eats. We can't formalize that for it depends on taking into account time. Or expand the simple present as in Example 45.

   n. $\forall x$ [(— is a person) $(x) \wedge \forall y$ ((— a dog) $(y) \to \neg$ (— likes —) (x, y))
   $\to \neg$ (— is a postman) (x)
   This is to take the "will" to indicate a general law.

   o. Not formalizable (compare Example 63).

   p. If you formalize this as "(— is in —) (Paris, France)" you'll have to give an explanation of how we can view locations as things and what it means for one location to be in another. See my *Time and Space in Formal Logic*.

   q. $2 + 2 = 4$
   Part of a formal theory.

   r. $7 + 8 = 15$.

   s. Not formalizable: "strong" is a relative adjective ("handsome" is OK because it is meant to apply only to men).

t. (— a man) (Dick) $\wedge$ (— is a student) (Dick)

u. $\neg$ (— likes —) (Dick, Puff)

v. Not formalizable because “homework” is a mass term.

w. This is ambiguous because of the new use of “they” as a singular indefinite pronoun. It could mean that the person will run quickly, or it could mean the dogs will run quickly. In either case it is not formalizable because of the adverb “quickly” and because it depends on time: when the person sees more than one dog. Also, can’t formalize “more than one” until Chapter 16.

x. $\exists x$ [(— is a square) (x) $\wedge$ (— is round) (x)]
This is to take squares to be things and “round” as an absolute adjective. We can ensure that this is not true in a model by formalizing it relative to a meaning axiom:
$\forall x$ [(— is a square) (x) $\rightarrow$ $\neg$ (— is round) (x)]

y. $\forall x$ [(— is an atom) (x) $\rightarrow$
$\exists y$ ((— has diameter —) (x, y) $\wedge$ (– < –) (y, 1/2 cm)]
This is to take atoms as things and measurements as things.

z. $\forall x$ [(— is a man) (x) $\rightarrow$ (— amuses —) (x, x)]
This is to view the exercise as a general rule, habitual. And it’s not to take “man” as meaning “person”.

aa. $\forall x$ [((— is a person) (x) $\wedge$ (— helps —) (x, x)) $\rightarrow$
(— helps —) (God, x)]
This is to take “them” as indicating a person (like “one”), “God” as a name of a thing and “— helps —” as meaning habitually.

bb. $\forall x$ [((— is a person) (x) $\wedge$ (— takes —) (x, Math 101))
$\rightarrow$ ((— passes —) (x, Math 101) $\vee$ (— fails —) (x, Math 101))]
This is to take courses as things and “will” as atemporal.

cc. Not formalizable. What one does is not a thing.

dd. Not formalizable: can’t formalize “dozen”.

ee. (— loves) (Suzy, Spot)
“even”, like “but” indicates some reason for surprise, and we can’t take account of that looking only at truth-values.

ff. (— loves —) (Suzy, Spot)
If you think that this and the last exercise should be distinguished, you’d need to add a meaning axiom governing “even”. Good luck.

gg. Not formalizable because it quantifies over collections.

hh. Not formalizable: we treat “exists” as a quantifier, not a predicate.

3. Two ways depending on whether we take the negation to be for the proposition as a whole or applied only to the predicate>

   i. $\exists x$ ((— is a cow) (x) $\wedge$ $\neg$ (— is white) (x))

   ii. $\neg$ $\exists x$ ((— is a cow) (x) $\wedge$ (— is white) (x))

   For (i) to be true, there must exist a cow. However, (ii) could be true if there were no cows. Our convention on negations says to take (i).

4. I separate the propositions in the inference with • .

a. • $\forall x$ [(— is a man) (x) $\rightarrow$
  ((— will be saved) (x) $\vee$ (— will be damned) (x))]
Therefore, • $\forall x$ [(— is a man) (x) $\rightarrow$ ((— will be saved) (x)]
  $\vee$ $\forall x$ [(— is a man) (x) $\rightarrow$ ((— will be damned) (x)]
Invalid. Could have a model in which some are saved and some are damned. We cannot take account of the tense, which means we are viewing the saving or damning of a person as an essential attribute.

b. • $\forall x$ [(— is a horse) (x) $\rightarrow$ (— is a mammal) (x)]
Therefore, • $\exists x$ (— is a mammal) (x)
Invalid. All does not imply exist.

c. • $\exists x$ [(— is a teacher) (x) $\wedge$ (— is nasty) (x)]
• $\exists x$ [(— is a teacher) (x) $\wedge$ (— is nasty) (x) $\wedge$ (— yells) (x)]
• $\forall x$ [(— is a teacher) (x) $\wedge$ (— is nasty) (x) $\wedge$ (— yells) (x)) $\rightarrow$
  $\exists y$ (— is a student) (y) $\wedge$ (— yells at —) (x, y)]
Therefore, • $\exists x$ [(— is a teacher) (x) $\wedge$
  $\exists y$ (— is a student) (y) $\wedge$ (— yells at —) (x, y)]
Relative to $\forall x$ [$\exists y$ (— yells at —) (x, y) $\rightarrow$ (— yells) (x)]

This is to read "yells at students" to mean yells at some students, since a teacher could hardly yell at all students throughout the world.
  This is valid.

d. • $\forall x$ [((— is a strawberry) (x) $\rightarrow$ (— is red) (x)) $\rightarrow$
  $\exists y$ ((— is a person) (y) $\wedge$ $\neg$ ((— cannot see — among leaves) (x, y)]
• $\forall x$ [((— is a strawberry) (x) $\rightarrow$ (— is red) (x))]
Therefore,
• $\exists y$ ((— is a person) (y) $\wedge$ $\neg$ ((— cannot see — among leaves) (x, y)]

This seems O.K., but there are two problems. First, I've used "— can see — among leaves" as an atomic predicate, though "— is a leaf" could also be atomic, for otherwise we'd have to talk about the location of the strawberries. Second, this takes no account of time, for a strawberry becomes red. The first premise should read "When strawberries are red, color blind people cannot see them among their leaves" and we have no way to formalize that temporal premise.

e. Not formalizable. The problem is that "fruit" is a mass term. We can't understand it's use in the second premise as meaning "all things that are fruit". Contrast with the use of "vegetables".

f. Not formalizable. The words "just", "fair", and "good" are not used as common nouns.

g. • (— is a student of —) (Suzy, Dr. E)
Therefore, • (— is a student) (Suzy)
  Relative to [$\forall x\, \forall y$ (— is a student of —) (x, y) $\rightarrow$ (— is a student) (x)]

The informal inference is valid, so we need a meaning axiom to ensure that the formalization is valid.

h. • $\forall x$ [(— is a child) (x) $\to$
$\forall y$ ((— is a mother of —) (y, x) $\to$ (— loves —) (x, y)]

• $\forall y$ [(— is a mother) (y) $\to$ $\exists x$ ((— is a child) (x) $\wedge$ (— has_p —) (y, x))]

Therefore, • $\forall y$ [(— is a mother) (y) $\to$ $\exists x$ (— loves —) (x, y)]

Relative to $\forall y$ [$\exists x$ (— is a mother of —) (y, x) $\to$ (— is a mother) (y)]

and $\forall y$ $\forall x$ [((— is a mother) (y) $\wedge$ (— is a child) (x) $\wedge$
(— has_p —) (y, x)) $\to$ (— is a mother of —) (y, x)]

The informal inference is valid. We interpret "his" as gender neutral because the premise says "every child". We do not include that a child has only one mother because that is not explicitly stated, even though "his" might be interpreted to mean only one. (Does "Dick feeds his dog" mean Dick has only one dog?) Anyway, we do not have the means to formalize uniqueness—that awaits Chapter 16. We need a meaning axiom relating "mother of" to "mother". Note also the formalization of the passive into active. I'll let you establish that the semi-formal inference is valid relative to the meaning axioms. (You thought this one would be easy?)

i. The obvious formalization is:
• $\forall x$ [(— is a fool) (x) $\to$ (— can solve —) (x, Exercise 1)]
• $\neg$ (— can solve —) (Dick, Exercise 1)
Therefore, • $\neg$ (— is a fool) (Dick)

But the semi-formal inference is valid, and the informal one is invalid.
The problem is that "any fool" is used hyperbolically, and the formalization cannot take account of that, only the truth-values. Note that the formal version takes exercises to be things.

j. • $\exists x$ [(— is a person) (x) $\wedge$ (— can solve —) (x, Exercise 1) $\to$
$\exists y$ ((— is a mathematician) (y) $\wedge$ (— can solve —) (y, Exercise 1)]

• (— is a mathematician) (Reginald) $\wedge$ $\neg$ (— can solve —) (Reginald, Exercise 1)

Therefore, • $\neg$ $\exists x$ [(— can solve —) (x, Exercise 1)]

Informal and formal inferences are both invalid.

k. • $\exists x$ [(— is a person) (x) $\wedge$ (— can solve —) (x, Exercise 1)] $\to$
$\forall y$ [(— is a mathematician) (y) $\to$ (— can solve —) (y, Exercise 1)]

• (— is a mathematician) (Reginald) $\to$ $\neg$ (— can solve —) (Reginald, Exercise 1)

Therefore, • $\neg$ $\exists x$ (— can solve —) (x, Exercise 1)

The formal and informal inferences are invalid. Perhaps a machine could solve Exercise 1.

l. • $\forall x$ [((— is a person) (x) $\wedge$ (— can solve —) (x, Exercise 1))
$\to$ (— is a mathematician) (x)]

• $\neg$ (— can solve —) (Ralph, Exercise 1)

Therefore, • $\neg$ (— is a mathematician) (Ralph)

The informal and semi-formal inferences are both invalid, for the same reason as the previous exercise, for Ralph is not a person.

m. • $\exists x$ [(— is a boy) $(x) \wedge$ (— likes —) (x, Ralph)]
• $\forall y$ [(— is a dog) $(y) \rightarrow \forall z$ ((— is a boy) $(z) \rightarrow$ (— likes —) (y, z))]
• (— is a dog) (Ralph)
Therefore, • $\exists x$ [(— likes —) (x, Ralph) $\wedge$ (— likes —) (Ralph, x)]
Valid.

n. Not formalizable because depends on taking account of time to show it is invalid.

o. Not clear that the inference even makes sense: "purple" is being used as an adjective in the first premise, but as the name of a color in the second. And the second premise takes colors to be things.

Chapter 15

Exercises p. 142

3. The interpretation of it is the same for all models, relative to the universe of the model.

Exercises p. 143

1. a. Each a is in one equivalence class, namely, $[a]$ because $a \simeq a$.
   Suppose now that $a \in [b]$ and $a \in [c]$. We need to show that $[b] = [c]$.
   Suppose that $d \in [b]$. We then have $a \simeq b$ and $d \simeq b$. So by symmetry and transitivity, $a \simeq d$. So since $a \simeq c$, we have $d \simeq c$. So $d \in [c]$.
   Similarly if $e \in [c]$, then $e \in [b]$. So for any d, $d \in [b]$ iff $d \in [c]$.
   Hence $[b] = [c]$.

   b. Clearly $a \simeq a$. And if $a \simeq b$, then $b \simeq a$.
   If $a \simeq b$, and $b \simeq c$, then all three are in the same subset, so $a \simeq c$.

   c. This comes straight from applying the definition of explicit identity.

Chapter 16

Exercises p. 153

1. a. $\forall x$ [(— is a person) $(x) \rightarrow$
($\exists y$ (— is a person) $(y) \wedge$ (— loves —) (x, y))]

b. $\forall x$ [(— is a person) $(x) \rightarrow$
($\exists y$ (— is a person) $(y) \wedge\ y \not\equiv x \wedge$ (— loves —) (x, y))]

c. Not formalizable. Can't quantify over times.

d. Choices. Can read this as "Everyone knows someone else who is famous", or can interpret it as requring a meaning axiom that no one knows himself, or can accept that a famous person knows himself or herself. I'll leave those to you.

e. (Ralph $\equiv$ Ralph) $\wedge\ \neg$ (— is a dog) (Ralph)

f. $\exists_{\geq 2}$ [(— is a cat) $(x) \wedge$ (— is nice) (x)]

g. $\neg\, \exists x$ [(— is a person —) $(x) \wedge$
$\exists y$ ((— is a logician) $\wedge$ (— likes —) (x, y))]
Note for this to be true in a model, any person who is a logician doesn't like herself or himself.

h. Not formalizable: "trouble" is a mass term. Can't formalize it even if we read it as "No one knows the troubles I've seen" because troubles are not things: try distinguishing them.

i. $\neg\, \exists x$ [(— is a person) $(x) \wedge\ \exists y$ ((— is a cat) $(y) \wedge$ (— kicks —) (x, y))
$\wedge$ (— is a philosopher) (x)]

j. $\forall x$ [(— is a person) $(x) \wedge\ \exists y$ ((— is a cat) $(y) \wedge$ (— kicks —) (x, y))
$\rightarrow\ \neg$ (— is a philosopher) (x)]
Note that (i) and (j) are equivalent semi-formal wffs.

k. $\neg\, \exists x$ [(— barks) $(x) \wedge\ \neg$ (— is a dog) (x)]

l. $\neg\, \exists x$ [((— is a person) $(x) \wedge$ (— is in New Mexico) (x))
$\wedge\ \forall y$ ((— is a person) $(y) \wedge$ (— is in Nevada) (y)
$\wedge$ (— is taller than —) (x, y))]
To take "New Mexico" and "Nevada" as names of things (places), we'd have to give an analysis of what it means for a thing to be in a location.You can see how to do that in my *Time and Space in Formal Logic*.

m. $\exists x$ [(— is a person) $(x) \wedge\ \forall y$ ((— is a person) $(y) \wedge\ y \not\equiv x$
$\rightarrow$ (— is taller than —) (x, y)

n. $\forall x$ [(— is a dog) $(x) \wedge$ (— is in Cedar City) $(x) \rightarrow$
(— is smarter than —) (Ralph, x)]
Note that this is false in a model in which "smarter than" is interpreted so that nothing is smarter than itself and Ralph is a dog.

o. (— is a dog) (Ralph) ∧ ∀*x* [(— is a dog) (*x*) ∧
(— is in Cedar City) (*x*) ∧ Ralph ≢ *x* →
(— is smarter than —) (Ralph, *x*)]

p. ∃*x* [(— is a woman) (*x*) ∧ (— sews) (*x*)]
∧ ¬ ∃*y* [¬ (— is a woman) (*y*) ∧ (— sews) (*y*)]

q. $\exists_{\geq 2}x$ [(— is a person) (*x*) ∧ ∀*y* ((— loves —) (*x*, *y*)
→ (— is a dog) (*y*)]

r. $\exists_{\geq 2}x$ [(— is a person) (*x*) ∧ ∃*y* ((— loves —) (*x*, *y*)
∧ (— is a dog) (*y*)]
∧ ¬ ∀*x* [(— is a person) (*x*) ∧ ∃*y* ((— loves —) (*x*, *y*)
∧ (— is a dog) (*y*))]

s. Not formalizable unless you can provide a meaning axiom for "couples".

t. $\exists!_4\, x$ [(— is a cat) (*x*) ∧ ∃*y* ((— is a tree) (*y*) ∧
(— is sitting in —) (*x*, *y*))]

u. ¬ ∃*x* ∃*y* ∃*z* [(— is a person) (*x*) ∧ (— is a person) (*y*) ∧
(— is a person) (*z*) ∧ *x* ≢ *y* ∧ *x* ≢ *z* ∧ *y* ≢ *z*
∧ (— lives in Cedar City) (*x*) ∧ (— lives in Cedar City) (*y*)
∧ (— lives in Cedar City) (*z*)
∧ ∃*w* ((— is a father of —) (*w*, *x*) ∧ (— is a father of —) (*w*, *y*) ∧
(— is a father of —) (*w*, *z*))]
Relative to ∀*x* ∀*y* [(— is a father of —) (*x*, *y*) → (— is a father) (*x*)]

v. $\exists_{\geq 33}x$ [(— is a cat) (*x*) ∧ ¬ ∃*y* ((— is a person) (*y*) ∧
(— scratched —) (*x*, *y*)]
∧ $\exists_{\leq 411}x$ [(— is a cat) (*x*) ∧ ¬ ∃*y* ((— is a person) (*y*) ∧
(— scratched —) (*x*, *y*)]

w. Not formalizable because it involves the notions of "may" and "can" in the sense of "possible", and it quantifies times.

x. It looks like this could be formalized, except for "can" which means "is possible". And "greater" is not clear. And it does not say that God can be conceived. All in all, this assumption about what "God" means is at best unclear.

y. ∀*x* ∀*y* [((— is a mind) (*x*) ∧ (— is a body) (*y*)) → *x* ≢ *y*]
This is formalized on the assumption that minds and bodies are things.
If they are not, then the wff is true by default.

3. I mark the propositions in the inference with • .

a. • (— wrote *Huckleberry Finn*) (Mark Twain)
• Mark Twain ≡ Samuel Clemens
Therefore, • (— wrote *Huckleberry Finn*) (Samuel Clemens)
Relative to ∀*x* [(— wrote *Huckleberry Finn*) (*x*) → (— wrote) (*x*)]
Valid.
You could take *Huckleberry Finn* to be the name of a thing, a book, but if

so it can't be any one copy of it. So what kind of thing is it? Not abstract, since at one time it didn't exist and now it does.

b. • $\exists ! x$ (— is a President of the U.S.) (x)
• (— is a President of the U.S.) (George Bush)
• George McGovern $\not\equiv$ George Bush
Therefore, • $\neg$ (— is a President of the U.S.) (George McGovern)
Valid.
This elides the difficulty of "President of the U.S." being really a descriptive name. You might take "the U.S." to be a name and argue that countries are things.

c. • (— is a dog) (Ralph) $\wedge$ (— is a dog) (Juney) $\wedge$ $\neg \exists x$ [(— is a dog) (x) $\wedge$ $\neg$ (Ralph $\not\equiv x$) $\wedge$ (Juney $\not\equiv x$)]
• (— barks) (Ralph) $\wedge$ (— barks) (Juney)
Therefore, • $\forall x$ [(— is a dog) (x) $\rightarrow$ (— barks) (x)]
Valid.

d. • Ralph $\not\equiv$ Juney
• $\neg$ (— is a puppet) (Juney)
Therefore, • (— is a puppet) (Ralph)
Invalid. Ralph could be a parakeet.

e. • $\exists !_{4{,}319}\, x$ [(— is a horse) (x) $\wedge$ (— lives in Utah) (x)]
• $\exists !_{18{,}317{,}271}\, x$ (— is a horse) (x)
Therefore,
• $\exists !_{18{,}312{,}956}\, x$ [(— is a horse) (x) $\wedge$ $\neg$ (— lives in Utah) (x)]
Relative to $\forall x\, \forall y$ [(— lives in Utah) (x, y) $\rightarrow$ (— lives) (x)]
Valid.

f. • $\forall x$ [(— is a father) (x) $\rightarrow$ $\exists_{\geq 2}\, y$ (— is a son of —) (y, x)]
• $\forall x\, \forall y\, \forall z$ [((— is a son of —) (y, x) $\wedge$ (— is a son of —) (z, x) $\wedge$ $y \not\equiv z$) $\rightarrow$ (— is a brother of —) (y, z)]
• $\forall x$ [(— is a man) (x) $\rightarrow$ (— is a son) (x)]
Therefore, • $\forall x$ [(— is a man) (x) $\rightarrow$ $\exists y$ (— is a brother of —) (x, y)]
Relative to $\forall y$ [$\exists x$ (— is a son of —) (y, x) $\leftrightarrow$ (— is a son) (y)]
and $\forall x$ [$\exists y$ (— is a brother of —) (y, x) $\leftrightarrow$ (— is a brother) (y)]
and $\forall x\, \forall y$ [(— is a brother of —) (y, x) $\rightarrow$ (— is a brother of —) (x, y)]
Valid.

g. • $\forall x$ [(— is a cat) (x) $\rightarrow$ (— has fleas) (x)]
• (— is a dog) (Juney)
• $\neg \exists x$ [(— is a dog) (x) $\wedge$ (— is a cat) (x)]
Therefore, $\neg$ (— has fleas) (Juney)
Invalid.
I can't see any way to formalize this to take into account that fleas are things.

h. This shows the problem with taking "nothing" to be a noun and not a quantifier.

Aristotelian Logic

Exercises p. 195

1.–5. See the definitions in the text.

6. Whether the claim is universal or particular.

7. Whether a claim is affirmative or negative.

8. Categorical. Universal affirmative. Subject: “dogs”. Predicate: “carnivore”.

9. Categorical. Particular negative. Subject: “cat”. Predicate: “carnivore”.

10. Categorical. Universal affirmative. Subject: “Tom”. Predicate: “basketball player”.

11. Categorical. Universal negative. Subject: “fire truck”. Predicate: “painted green”.

12. Categorical via rewrite as “All donkeys are meat eaters”. Universal affirmative. Subject: “donkeys”. Predicate: “meat eaters”.

13. Categorical affirmative via rewrite: “Some chimpanzee is a thing that can communicate by sign language”. Particular affirmative. Subject: “chimpanzee”. Predicate: “a thing that can communicate by sign language”.

14. Categorical affirmative via rewrite “All border collies are sheep chasers”. Universal affirmative. Subject: “border collies”. Predicate: “sheep chasers”.

15. Categorical affirmative via rewrite “No one who knows critical thinking is a thing that will starve”. Universal negative. Subject: “one who knows critical thinking”. Predicate: “thing that will starve”.

16. Not categorical. A disjunction of categorical claims.

17. Categorical via rewrite “All heroin addicts are things that cannot function in a nine-to-five job”. Universal affirmative. Subject: “heroin addicts”. Predicate “things that cannot function in a nine-to-five job”.

18. Categorical. Particular affirmative. Subject: “people who like pizza”. Predicate: “vegetarians”.

19. Categorical via rewrite “Some canary is a thing that cannot sing”, but justifying that is not trivial. Universal negative. Subject: “canary”. Predicate: “a thing that cannot sing”.

20. Categorical via rewrite “Socrates was not a thing that has a cat”. Universal negative. Subject: “Socrates”. Predicate: “a thing that has a cat”.

21. Not categorical. A conditional.

22. There is no obvious rewrite of this that is categorical.

Exercises p. 197

1. a. They both can’t be true, and they both can’t be false.
   b. They both can’t be true.
   c. They both can’t be false.
2. “All dogs bark” rewritten as “All dogs are things that bark.”
   They can’t both be true. If “all” has existential import, then they could both be false, so they are contraries. If no existential import, they are contradictories.

3. Contraries.
4. Contraries.
5. They can't both be true. But if "all" has existential import, they could both be false, so they are contraries. If "all" does not have existential import, they are contradictories, as the diagram says.
6. None.
7. Subcontraries.
8. None.
9. Can't both be true. If "all" has existential import, then both could be false, so they are contraries. If "all" has no existential import, they can't both be false, so they are contradictories.
10. Rewrite "Homeless people don't like to sleep on the street" as "All homeless people don't like to sleep on the street". Can't both be true. If "all" has existential import, they both could be false, so they are contraries. If "all" has no existential import, then they can't both be false, so they are contradictories.

<u>Exercises p. 199</u>

1. See the definition, p. 198.
2. Yes. Example:
   (a) All dogs are domesticated.
   Some dogs are not domesticated.
   Therefore, Cats meow.
   Valid because there is no way for both premises to be true.
   (b) All dogs are domesticated.
   All cats are felines.
   Therefore, all cats are felines.
   Valid.
3. Rewriting: All students at this school are people that pay tuition. Some people who pay tuition at this school are people who will fail. So some students at this school are people who will fail.
   Major term: "people who will fail". Minor term: "students at this school".
   Middle term: "people who pay tuition".
   Not in standard form, which is:
   Some people who pay tuition at this school are people who will fail.
   All students at this school are people that pay tuition
   So some students at this school are people who will fail.
   IAI
4. Rewriting:
   No wasps are things that will not sting.
   Some bumblebees are things that will not sting.
   Therefore, Some bumblebees are not wasps.
   Major term: "wasps". Minor term: "bumblebees". In standard form.
   EOO

5. Rewriting:
    No pacifist is a person who will fight in a war.
    Arf is a pacifist.
    Therefore, Arf is not a person who will fight in a war.
    Major term: "a person who will fight in a war". Minor term: "Arf".
    In standard form.
    EAA
6. Rewriting:
    No badly managed business is profitable.
    No oyster cultivating business in North Carolina is badly managed.
    Therefore, Some oyster cultivating business in North Carolina is profitable.
    Major term: "profitable". Minor term: "oyster cultivating business in North Carolina".
    In standard form. EEI
7. Rewriting:
    No thing that is smarter than a dog is a thing that coughs up hairballs.
    All cats are things that cough up hairballs.
    Therefore, No cat is smarter than a dog.
   Major term: "smarter than a dog". Minor term: "cat".
   In standard form. EAE.
8. Not a categorical syllogism.
9. Rewriting:
    All police chiefs who interfere with the arrest of city officials are people who are fired.
    All people who are fired are people who collect unemployment insurance.
    Therefore, Some police chiefs who interfere with the arrest of city officials are people who collect unemployment insurance.
   Major term: "people who collect unemployment insurance". Minor term: "police chiefs who interfere with the arrest of city officials".
   Not in standard form. Reverse the order of the premises. AAI.
10. Not a categorical syllogism.
11. This can be made into two syllogisms.
    All that is beautiful is good.
    All beautiful people are beautiful.
    Therefore, all beautiful people are good.
   Major term "good". Minor term: "beautiful people".
   Form: AAA.
   The second syllogism uses the conclusion of the first one.
    All that is good is loved by the gods.
    All beautiful people are good.
    Therefore, All beautiful people are loved by the gods.
   Major term: "loved by the gods". Minor term: "beautiful". In standard form.
   AAA

12. Rewriting:

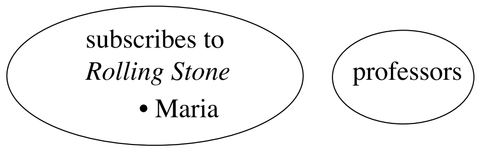
    All steak is meat.
    All meat is loved by dogs.
    Therefore, All steak is loved by dogs.

    Not in standard form; reverse the order of the two premises.
    AAA.

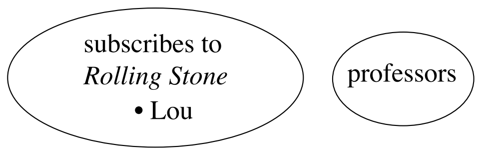
<u>Exercises p. 202</u>

1. Invalid. Lee could be one of the ones who does attend lectures.
   Not every ≠ every not.

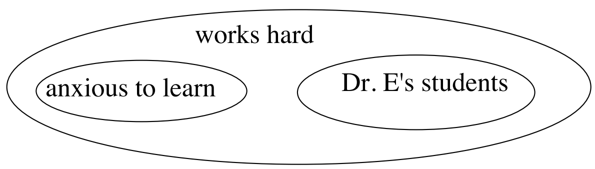
2. Invalid



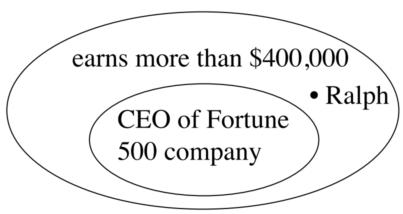
3. Valid



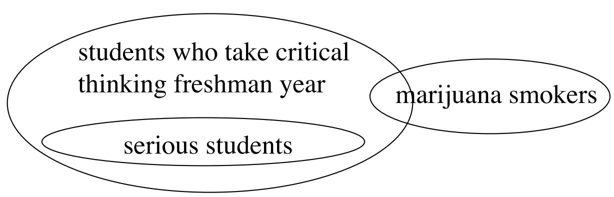
4. Invalid.



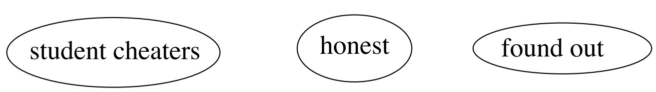
5. Invalid



6. Invalid.



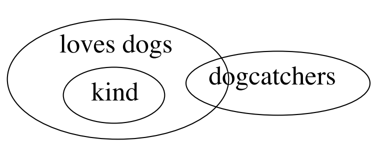
7. Invalid.



8. Invalid. George could be mute.

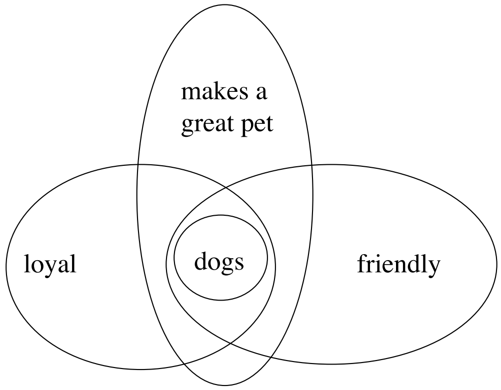
9. Valid.

10. Invalid.



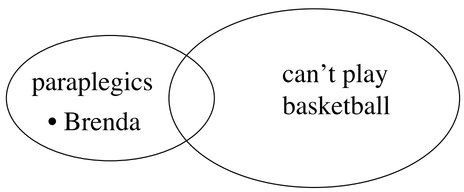
11. Invalid. The premise is *not* “all hogs grunt.” Don’t mistake your knowledge of the world for what’s actually been said. It’s reasoning in a chain with “some.”

12. Valid



13. Invalid. Also not an Aristotelian syllogism.

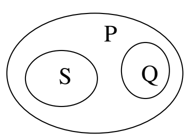
14. Invalid



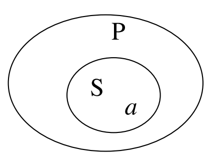
15. Valid, but can’t show it as an Aristotelian syllogism because need to look at the internal structure of “hate cats” and “hate mammals”. The first premise is irrelevant.

16. Valid. The direct way of reasoning with no. If you think the conclusion is false, which premise isn’t true?

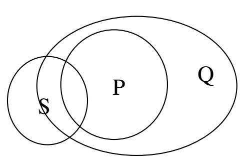
17. Invalid.



18. Valid. Only possible picture:

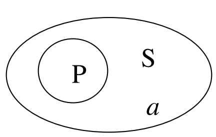


19. Valid. Must have:

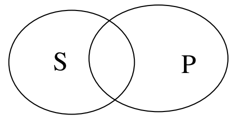


(S could be entirely within P or Q, but still the conclusion would be represented as true.)

20. Invalid.



21. Invalid.



22. Valid.