Propositional Logics

The Semantic Foundations of Logic

THIRD EDITION

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Dedicated to

Peter Eggenberger
Harold Mann
and
Benson Mates

with gratitude for their encouragement and guidance

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Summary of Logics

Preface

If logic is the right way to reason, why are there so many logics?

Viewing logics as formalizations of how we do or should reason, we can find a structural and conceptual unity based on common assumptions about the relation of language, reasoning, and the world. What we pay attention to in reasoning determines which logic is appropriate.

In order for you to understand this I have retraced my steps: from the concrete to the abstract, from examples to general theory, and then to reflections on the significance of the work. In doing so I have had to begin at the beginning: What is logic? What is a proposition? What is a connective? If much seems too well known to be of interest, then plunge ahead. The chapters can be read more or less independently, which explains the occasional repetitions.

Chapter I is devoted to assumptions about the nature of propositions and what forms of propositions we will study. In Chapter II we then have the simplest symbolic model of reasoning we can devise given those assumptions. In classical logic a proposition is abstracted to only its truth-value and its form, relative to the propositional connectives. This provides a standard of reference for other logics.

In Chapter II I also present a Hilbert-style formalization of the notion of proof and syntactic deduction that I use throughout the book. The metalogical investigations that I concentrate on concern the relation between the semantic and syntactic notions of consequence, and whether or how those can be represented in terms of theorems or valid formulas by means of a deduction theorem.

Chapter III sets out the simplest example of a logic that incorporates some aspect of propositions other than truth-value into the semantic analysis. Taking the subject-matter of a proposition to be a primitive notion, we get the archetype of how to incorporate differing aspects of propositions into semantics.

Following Chapter III is a Summary and Overview which serves as an introduction to all that follows. The succeeding chapters present examples of many different logics based on differing semantic intuitions all of which can be understood within a general framework that is presented in Chapter IX. That framework arises from the view that each logic, except for classical logic, incorporates into the semantics some aspect of propositions other than truth-value and form. Each logic analyzes an ‘if . . . then . . .’ proposition classically if the aspects of antecedent and consequent are appropriately connected, while rejecting the proposition otherwise. As we vary the aspect, we vary the logic. I have argued for this bivalent falsity-default analysis of semantics throughout this volume, in part by presenting a wide variety of logics in that form, and I have used that analysis further in *Epstein, 1992* and *Epstein, 2012A.*

The general form of semantics is not intended to replace other semantics. For example, under certain assumptions possible-world semantics are a good explanation of the ideas of modal logics. But providing uniform semantics that are in reasonable conformity with the ideas on which various logics are based allows for comparisons and gives us a uniform way in which to approach the sometimes overwhelming multiplicity of logics.

In particular, the overview of the general framework allows Stanisław Krajewski and I to consider in Chapter X the extent to which one logic or way of seeing the world can be reduced to another. We present a general theory of translations and try to characterize what we mean when we say that a translation preserves meaning.

The semantic framework I set out in Chapter IX is a very weak general form of logic that becomes usable only upon the choice of which aspect of propositions we deem significant. But then is logic relative to the logician? Or does a notion of necessary truth lie in this general framework? In Chapter XI I discuss how our agreements about how we reason determine our notion of objectivity.

Throughout I have tried to find and then make explicit those assumptions on which our reasoning and logic are based. I have repeated the statement of certain of those assumptions in different places, partly because I want the chapters to be as self-contained as possible but also because it is important to see those assumptions and agreements in different contexts and applied differently to be able to grasp their plausibility and pervasiveness.

What I am doing here can be seen as founding logic in ordinary language and reasoning. When nonconstructive assumptions are used to apply mathematics to logic to prove theorems about our formalizations we can see precisely where they are needed. Those assumptions I treat as abstractions from experience. However, they need not be viewed that way, and I have attempted to provide alternate readings of the technical work based on the view that abstract things such as propositions are as real or more real than the objects we daily encounter. Most of the discussion of these matters is in Chapter I and in the development of classical logic in Chapter II, particularly Section II.G. In Chapter IX I point out specific nonconstructive, infinitistic abstractions of the semantics that we usually make in pursuing metalogical investigations. This general approach to modeling and theories is explored more fully in my essays in *Reasoning in Science and Mathematics*, while the issue of whether logic is prescriptive or descriptive is explored in my book *Prescriptive Reasoning.*

I have included many exercises, some of them routine, many requiring considerable thought, and some which are open questions (marked ‘Open’).

Depending on the choice of which are assigned, this book can serve as a text in an undergraduate course, a text for a graduate course, or as the basis for research.

There are important subjects in the study of propositional logics that I do not deal with here. I have not discussed the algebraic analyses of propositional logics, for which you can consult *Rasiowa, 1974* and *Blok and Pigozzi, 1989.* I have made no attempt to connect this work with the categorial interpretation of logic, for which you can consult *Goldblatt, 1979.* Nor have I dealt with other approaches to the notion of proof in propositional logics. And there are many propositional logics I have not discussed, quite a few of which are surveyed also in *Marciszewski, 1981* as well as in *Haack, 1974,* and *Gabbay and Guenthner, 1989,* which also discuss philosophical issues.

This is not the story of all propositional logics. But I hope to have done enough to convince you that it is a good story of many logics that brings a kind of unity to them.

> In the discussions of the wise there is found unrolling and rolling up,
> convincing and conceding; agreements and disagreements are reached.
> And in all that the wise suffer no disturbance. —Nagasena

Come, let us reason together.

Preface to the third edition

In 1992 I was asked to publish *Predicate Logic*, the second volume of this series *The Semantic Foundations of Logic*. I suggested also doing a second edition of *Propositional Logics*. There were a few corrections that colleagues had pointed out, and I thought I could clean up the text a bit. It turned out that a lot of corrections were needed, both to the technical work and the exposition. For that edition I revised the entire text, with more changes than I could easily list here. Among the most significant are the correction or simplification of many axiomatizations, the addition of examples of formalization of ordinary reasoning, and the addition of exercises to make the text more suitable for individual or classroom use.

In 2011 Esperanza Buitrago-Díaz came to the Advanced Reasoning Forum at Dogshine as an ARF Student Fellow to work through the second edition of this text with me. Her questions and comments, difficulties and insights led me to prepare this new edition. The most notable differences from the second edition are:

- The chapter on the general framework now follows the development of the examples of logics rather than preceding them.

- In the chapter on modal logics the logic of logical necessity is developed before accessibility relations are introduced
- In the chapter on paraconsistent logics a new approach to paraconsistency is introduced by modifying the notion of semantic consequence.

In my recent studies I have tried to place formal logic in the larger context of a general theory of inference. The first presentation of that was in my *Five Ways of Saying* "*Therefore*". The mature version can be found in my series of books *Essays on Logic as the Art of Reasoning Well*. It would have been too large a project to modify this text to fully take account of that work, although I have made some changes in Chapters I and II to reflect those ideas.

There is, after all, no end but only a continual beginning.